

Isentrope

$$dQ = dU + pdV = n c_v dT + pdV = 0 \Rightarrow n c_v dT = -pdV \quad (1)$$

$$dQ = dH - Vdp = n c_p dT - Vdp = 0 \Rightarrow n c_p dT = Vdp \quad (2)$$

$$\frac{(2)}{(1)} \Rightarrow \alpha = - \frac{Vdp}{pdV} \Rightarrow \frac{dp}{p} = -\alpha \frac{dV}{V} \quad (*)$$

Annahme: $\alpha(T) = \text{const.}$

$$\ln \left(\frac{p_2}{p_1} \right) = -\alpha \ln \frac{V_2}{V_1} = \alpha \ln \frac{V_1}{V_2} = \ln \left(\frac{V_1}{V_2} \right)^\alpha$$

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2} \right)^\alpha \Rightarrow p_1 V_1^\alpha = p_2 V_2^\alpha = \text{const} \quad (\text{Adiabaten-Gleichung})$$

$$\Rightarrow p_2 = (p_1 V_1^\alpha) \cdot \frac{1}{V_2^\alpha} \quad (\alpha > 1!)$$

$$\text{const} = \frac{pV}{T} = 1 \quad \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \Rightarrow \frac{p_2}{p_1} = \frac{T_2}{T_1} \frac{V_1}{V_2} \rightarrow \frac{T_2}{T_1} \left(\frac{V_1}{V_2} \right) = \left(\frac{V_1}{V_2} \right)^\alpha$$
$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{\alpha-1}{\alpha}} \quad \Rightarrow \quad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\alpha-1}$$

$$Q_{12} = 0$$

$$W_{12} = - \int_{V_1}^{V_2} p dV = - p_1 V_1^\alpha \int_{V_1}^{V_2} \frac{dV}{V^\alpha}$$

$$= \frac{p_1 V_1}{\alpha - 1} \left[\left(\frac{V_1}{V_2} \right)^{\alpha - 1} - 1 \right]$$

$$\stackrel{V \sim T}{=} \frac{n R_c T}{\alpha - 1} \left[\frac{T_2}{T_1} - 1 \right] = \frac{n R_c}{\alpha - 1} (T_2 - T_1) = n \overline{c}_V \Big|_{T_1}^{T_2} (T_2 - T_1) = U_2 - U_1$$

$$W_{t12} = H_2 - H_1 = n \overline{c}_p \Big|_{T_1}^{T_2} (T_2 - T_1) = n \frac{\overline{c}_p \Big|_{T_1}^{T_2}}{\overline{c}_V \Big|_{T_1}^{T_2}} \overline{c}_V \Big|_{T_1}^{T_2} (T_2 - T_1) = \overline{\alpha} \Big|_{T_1}^{T_2} W_{12}$$

$$\overline{\alpha} \Big|_{T_1}^{T_2} = \frac{\int_{T_1}^{T_2} \overline{c}_p(T) dT}{\int_{T_1}^{T_2} \overline{c}_V(T) dT} = \frac{\int_{T_1}^{T_2} \alpha(T) \overline{c}_V(T) dT}{\int_{T_1}^{T_2} \overline{c}_V(T) dT}$$