Trapezregel

$$I_T = \sum_{i=0}^{n-1} \frac{h}{2}\left(f(x_i) + f(x_{i+1})\right) = \frac{h}{2}\left(\sum_{i=0}^{n-1} f(x_i) + \sum_{i=0}^{n-1} f(x_{i+1})\right)$$

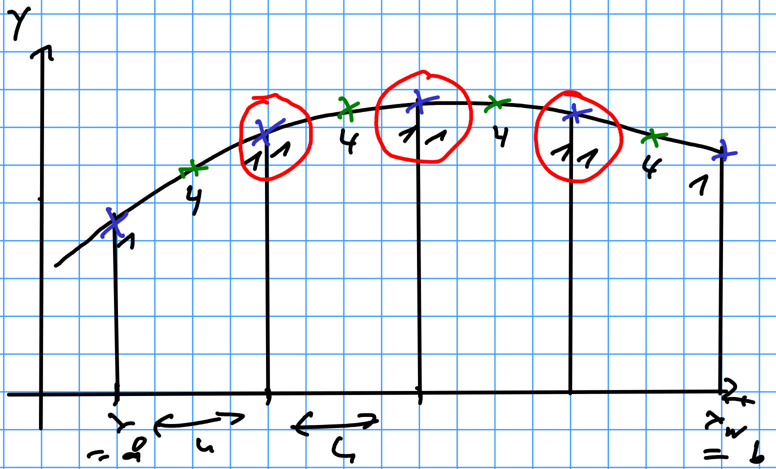
$$= \frac{h}{2}\left(f(x_0) + f(x_1) + \dots + f(x_{n-1}) + f(x_1) + \dots + f(x_{n-1}) + f(x_n)\right)$$

$$= \frac{h}{2}\left(f(a) + 2\sum_{i=1}^{n-1} f(x_i) + f(b)\right)$$

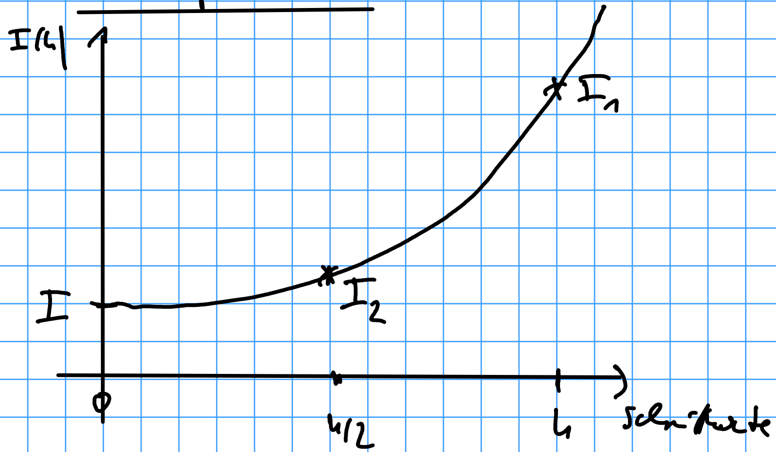
Simpsonregel

$$I_h = \frac{h}{6}\left(f(x_i) + 4f\left(x_i + \frac{h}{2}\right) + f(x_i + h)\right)$$

$$I_S = \frac{h}{6}\left(f(a) + 2\sum_{i=1}^{n-1} f(x_i) + 4\sum_{i=0}^{n-1} f\left(x_i + \frac{h}{2}\right) + f(b)\right)$$



Extrapolation



$$I(h) = I + ah^4 + O(h^6)$$

$$y = I + ax^4$$

$$I_1 = I + ah^4 \qquad | \cdot (-1)$$

$$I_2 = I + a\left(\frac{h}{2}\right)^4 \qquad +$$

$$= I + \frac{a}{16} h^4 \qquad | \cdot 16$$

$$16 I_2 - I_1 = 15 I$$

$$\Rightarrow \quad I = \frac{1}{15}\left(16 I_2 - I_1\right) + O(h^6)$$

Adaptive Integration am Beispiel

$\delta = 0.02$

l	r	I_1	I_2	Kommentar	
0.2	1.0	0.4377	0.5891	halbieren	
0.2	0.6	0.3168	0.3775	halbieren	
0.6	1.0	0.2723	0.2723	ok	+
0.2	0.4	0.2494	0.0598	halbieren	
0.4	0.6	0.1280	0.1393	ok	+
0.2	0.3	0.0254	0.0848	halbieren	
0.3	0.4	0.0345	0.0458	ok	+
0.2	0.25	0.0445	0.0355	ok	+
0.25	0.3	0.0403	0.0391	ok	+

$$I = \frac{1}{15}\left(16 I_2 - I_1\right)$$

0.5350 (0.5359)