Berechnung der natürlichen Splines

$x_i = [1,2,3,4,5]$ $\quad y_i = [1,2,1,1,4]$

$h_k = 1 \quad k = 1,..4$

Gleichungen für M_k:

$$M_k + 4M_{k+1} + M_{k+2} = -3y_k + 3y_{k+2} \qquad k=1,..N-2$$

$k=1$: $M_1 + 4M_2 + M_3 = 0$

$k=2$: $M_2 + 4M_3 + M_4 = -3$

$k=3$: $M_3 + 4M_4 + M_5 = 9$

nat. Sp. 1: $2M_1 + M_2 = 3$

nat. Sp 2: $M_4 + 2M_5 = 9$

$$\begin{pmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 4 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -3 \\ 9 \\ 9 \end{pmatrix}$$

naiv: $O((4N)^3) \sim 64N^3$

gut: $O(N^2)$

z.B.	$N=10$	100	64000
	$N=100$	10000	64 Mio

Bestimmung der Polynome

$$P_k(x) = (3-2s_k)\,s_k^2\,y_{k+1} + (1+2s_k)(1-s_k)^2\,y_k + s_k^2(s_k-1)\,M_{k+1} + s_k(s_k-1)^2\,M_k$$

$P_1(x)$: $\quad s_1 = x-1 \qquad y_1 = 1,\ y_2 = 2,\ M_1,\ M_2$

$P_2(x)$: $\quad s_2 = x-2 \qquad y_2 = 2,\ y_3 = 1,\ M_2,\ M_3$