Mehrdimensionales Newton-Verfahren

Bsp: $\vec{F}(\vec{x}) = \begin{pmatrix} F_1(x,y) \\ F_2(x,y) \end{pmatrix} = \begin{pmatrix} x^2+y^2-6 \\ x^3-y^2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\qquad \vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$

Startwert $\vec{x}_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

1d: $f(x_0+h) = f(x_0) + f'(x_0)\cdot h + \frac{1}{2} f''(x_0)\cdot h^2 + \frac{1}{3!} f'''(x_0) h^3 + \dots$

$\approx f(x_0) + f'(x_0)\cdot h \overset{!}{=} 0$

nd: $F(x_0+h) \approx F(x_0) + \left(\frac{\partial F}{\partial x}\right)_{x_0} \cdot h$ mit $\left(\frac{\partial F}{\partial x}\right)_{x_0} = \begin{pmatrix} \left(\frac{\partial F_1}{\partial x_1}\right)_{x_0} & \dots & \left(\frac{\partial F_1}{\partial x_n}\right)_{x_0} \\ \vdots & & \\ \left(\frac{\partial F_n}{\partial x_1}\right)_{x_0} & \dots & \left(\frac{\partial F_n}{\partial x_n}\right)_{x_0} \end{pmatrix}$

Wert x_k $\qquad x_0+h \to x \qquad x_0 \to x_k$

$F(x) \approx F(x_k) + \left(\frac{\partial F}{\partial x}\right)_{x_k} \underbrace{(x - x_k)}_{z} = 0$

$\Rightarrow \left(\frac{\partial F}{\partial x}\right)_{x_k} \cdot z = -F(x_k) \qquad \Rightarrow z = x - x_k \Rightarrow x = x_k + z$

$= x_{k+1}$

Beispiel:

$$F(\vec{x}) = \begin{pmatrix} x^2+y^2-6 \\ x^3-y^2 \end{pmatrix} \qquad \left(\frac{\partial F}{\partial x}\right) = \begin{pmatrix} 2x & 2y \\ 3x^2 & -2y \end{pmatrix}$$

Startwert: $\vec{x}_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad F(\vec{x}_0) = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \qquad \left(\frac{\partial F}{\partial x}\right)_{\vec{x}_0} = \begin{pmatrix} 2 & 2 \\ 3 & -2 \end{pmatrix}$

$$\begin{pmatrix} 2 & 2 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad \Rightarrow \quad z = \begin{pmatrix} 0.8 \\ 1.2 \end{pmatrix} \quad \Rightarrow \quad \vec{x}_1 = \begin{pmatrix} 1.8 \\ 2.2 \end{pmatrix}$$