

Untere Dreiecksmatrix :
$$\begin{pmatrix} 1 & 0 & 0 \\ f_2 & 1 & 0 \\ f_1 & f_3 & 1 \end{pmatrix} = L \quad L \cdot U = P \cdot A$$

Lösen von $Ax = b$:
$$L \underbrace{Ux}_y = PAx = Pb = \tilde{b}$$

Vorrücksubstitution:

$$\begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \\ \tilde{b}_3 \end{pmatrix}$$

$$y_1 = \tilde{b}_1$$

$$l_{21} y_1 + y_2 = \tilde{b}_2 \Rightarrow y_2 = \tilde{b}_2 - l_{21} y_1$$

$$l_{31} y_1 + l_{32} y_2 + y_3 = \tilde{b}_3 \Rightarrow y_3 = \tilde{b}_3 - l_{31} y_1 - l_{32} y_2$$

Rückwärts substitution

$$\begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$u_{33} x_3 = y_3 \Rightarrow x_3 = y_3 / u_{33}$$

$$u_{12} x_2 + u_{23} x_3 = y_2 \Rightarrow x_2 = (y_2 - u_{23} x_3) / u_{12}$$

$$u_{11} x_1 + u_{12} x_2 + u_{13} x_3 = y_1$$

$$\Rightarrow x_1 = (y_1 - u_{12} x_2 - u_{13} x_3) / u_{11}$$