

Using Applets for Physics Education- Case Study “Nonlinear Systems and Chaos”

by

Peter Junglas*

FHWT-Report No. I-2003-02

Vechta/Diepholz
February 2003

* Prof. Dr. Peter Junglas, Professor for Computer Science and Physics, FHWT Vechta/
Diepholz

Published by:

Private Fachhochschule für Wirtschaft und Technik Vechta/Diepholz
– Fachhochschule und Berufsakademie (FHWT)

Standort Vechta:
Rombergstr. 40
49377 Vechta
Tel. (04441) 915-0

Standort Diepholz:
Schlesierstr. 13 a
49356 Diepholz
Tel. (05441) 992-0

Standort Oldenburg:
Donnerschweer Straße 184
26123 Oldenburg
Tel. (0441) 34092-0

Abstract

Simulations with Java applets can be a reasonable tool to assist in physics education, supplementing theoretical lectures and classical experiments. For this purpose the concepts and graphical interfaces of the applets have to be designed according to didactic needs. This understood they make possible to include modern topics into beginner's courses, which usually can not be presented due to their mathematical complexity. This is exemplified with applets supporting a short course on basic phenomena of chaotic systems. They simulate the free and the harmonically excited mathematical pendulum, displaying space-time and phase space diagrams. Further applets allow to study Poincare sections or the dependence on initial conditions. Besides the presentation of qualitative features like basic motion types these applets can be used as "virtual experiments" leading to quantitative results. For each applet its design and function are explained together with its purpose during the course considered here.

Contents

Abstract	2
1.Introduction	4
2.Simulations with Applets for Physics Education	4
3.Comparison with Different Approaches	5
4.The Course „Nonlinear Systems and Chaos“	6
5.Basic Structure of the Applets	8
6.Description and Usage of Selected Applets	10
6.1.Mathematical Pendulum	10
6.2.Representation in Phase Space	11
6.3.Driven Pendulum	12
6.4.Driven Pendulum in Phase Space	13
6.5.Poincare Section	14
6.6.Dependence on the Initial Conditions	15
7.Conclusions	16
8.References	17
9.Biography	19

Illustrations

Model of a Duffing oscillator	8
Applet „Mathematical Pendulum“	9
Applet „Phase Space Representation“	12
Applet „Driven Pendulum“	13
Applet „Driven Pendulum in Phase Space“	14
Applet „Poincare Section“	15
Applet „Initial Conditions“	16

1. Introduction

Simulation is now well-established as a valuable tool for finding physical knowledge adding to the classical methods of experiment and theory. It uses numerical computation to find the solutions of complicated equations, which are not amenable to analytical methods. In this way it connects theory and phenomenology and provides means for a better understanding of the consequences of fundamental theories.

Theory and experiment have always been used in teaching physics to engineers: The classical lecture provides an overview of the phenomena and the basics of theory, supplementing hands-on courses or laboratories acquaint the students with experimental techniques and experiences. But simulation can be used profitably as well: It can serve as a didactic tool that allows to study the consequences of a theory in cases where the students lack the necessary mathematical skills. Furthermore it allows to imitate experiments, which can be used to get quantitative results with a computer under idealized conditions.

The didactic advantages of such “virtual experiments” are obvious: In contrast to real laboratory experiments all disturbing influences are eliminated from the beginning, the “experiments” always succeed and the physical relation that should be found is clearly apparent. Furthermore due to the complete reproducibility all students get identical results. Especially for small universities there is an additional financial aspect: Often the equipment of physics laboratories is rather limited in number and restricted to simple basic experiments, so that student activities have to be replaced by demonstration experiments during the lecture. Already available computers can remedy this situation by allowing the students to play an active role again in attaining physical insights.

Of course simulations can't replace real experiments in physics education. The difficulties, which every experimenter faces, and the care and accuracy necessary to cope with them are an integral part of the experiences students have to make. Furthermore the real world contact with the system to study leads to completely different activities and sensations than working with a computer, thereby substantially improving the learning success. But the largest threat in using simulated experiments is that the students don't differentiate between simulation and reality. Considering in particular the present general tendency towards virtualisation, where the borderline between “ego shooters” and “virtual warfare” is obscured purposely, the teacher has to clearly point out, which kind of knowledge can be gained using simulations – and which cannot!

2. Simulations with Applets for Physics Education

To implement the numerical algorithms that are needed for a simulation one can use all of the common programming languages and a lot of mathematical problem solving environments. In the project described here the well established Java language [1, 2] was used to create applets which have several advantages for the implementation of virtual experiments:

- They are easily integrated into HTML pages, which contain instructions or

additional information,

- they are independent of the type of computer or operating system,
- needed viewers (browser plugins) are widely and freely available [3],
- they can be used directly on the internet or on CD-ROMs.

From a programmer's point of view Java applets have further benefits:

- language constructs for time controlled actions and animations [4],
- standard libraries for graphical user interfaces [5],
- freely available classes for numerics, e. g. linear algebra, complex numbers and special mathematical functions [6],
- free development environments supporting a graphical programming style [7].

On the other hand one has to concede that a huge programming effort is necessary for the applets presented here. A possible remedy lies in the consequent application of reusable building blocks like JavaBeans [8], which can be assembled graphically to make up a complete applet. For this purpose the class library PhysBeans [9] has been used that is the outcome of several such projects. To further propagate such techniques it would be sensible to start an open source project which aims at the common development of such a basic library, for which PhysBeans might be a good starting point.

Applets forming virtual experiments can be integrated into physics education in several ways: They can be used in a lecture for demonstration of physical phenomena or for guided experiments in student's exercises. Since they can easily be made available through the internet, they are useful for home work after the lecture or even for more playful own explorations.

An increasing number of new physics textbooks contains such applets on a supplementing CD-ROM [10,11]. An early version of the PhysBeans library has been used in [11] for this purpose. The internet is a treasure trove of applets for all branches of physics. Good starting points are f. i. the link collections of PhysicsWeb [12], J. Loviscach [13] or the project „physik multimedial“ [14]. Especially in the area of non-linear oscillations and chaotic systems there exist fine applets [15, 16], which complement and largely extend the examples described below.

3. Comparison with Different Approaches

To solve nonlinear differential equations one can easily use general purpose numeric programs like Matlab [17], which allow to compute and plot the solution with a few high level commands. Taking examples from mathematics and physics courses it is shown in [18] how Matlab can quickly lead to solutions of mathematical problems coming from very different areas. That even algebra systems like Maple [19] can be used for this purpose has been exemplified in [20] with a case study of the mathematical pendulum. Though the focus of this paper is on the analysis of the equations using analytical tools like series expansion Maple is used to find numerical solutions as well.

For the application of such programs the students need basic knowledge not only of their operation, but also of the underlying mathematical problems. The main object of these studies are not the physical phenomena but the equations to describe them and the methods for their solution. For this reason this approach is better suited for advanced students which have the necessary

prerequisites in mathematics and programming. In this case it is an invaluable tool to combine aspects of mathematics, physics and computer science in an interdisciplinary way.

In contrast the task of the applets described here is to screen the students from the underlying mathematics by providing a graphical user interface for the simulation of an experimental situation. This allows to use them for freshmen physics courses, but makes them only applicable to the concrete physical context they are made for.

Certainly one can construct graphical user interfaces in Matlab as well, but this needs a much larger programming effort compared to the simple Matlab functions. Furthermore these programs need a Matlab environment which usually is not available to the students. Finally they can not be integrated into an HTML page in a simple way.

A completely different approach are the “virtual laboratories” presented in [21]: These are real experiments that can be remotely controlled via the internet including a real-time presentation of the results. A sophisticated reservation system prohibits simultaneous access by different users. Even though the fixed experimental setup restricts the manipulation possibilities (as well as the possibilities to make mistakes!) one gets the strong feeling of doing a real experiment. Simultaneously transferred video images of the experiment add to this impression.

To construct such a remotely controlled laboratory is rather expensive, therefore this will be done probably only for particularly interesting and complicated experiments, which justify the effort by leading to a high utilization over time, not for simple physical standard situations. The experiment described in [22] as part of a mechatronics course has the further advantage that here the actuators and sensors needed for the remote control are a matter of investigation themselves. In this case not only the proper experiment, but its construction in the form of a virtual laboratory is a useful project for the course.

4. The Course „Nonlinear Systems and Chaos“

Students of mechanical engineering at the FHWT have to attend two physics courses with 40 hours of lectures per semester. The complete first and the first half of the second semester are filled up with standard topics (oscillations and waves, acoustics, optics, special relativity, quantum mechanics, nuclear physics), the rest is used for special short courses of 6 to 8 hours each. The aim of these courses is to introduce some modern physics concepts which are not included in standard textbooks. The inclusion of current physics themes helps to motivate engineering students, buzzwords like 'general relativity' or 'chaos' arise their curiosity. Additionally these topics can have relations to main engineering courses: Basic principles of statistical mechanics prepare the ground for thermodynamics, the investigation of fields is helpful to understand the velocity field of fluid dynamics.

Under these aspects an introduction to nonlinear and chaotic systems is especially well suited: On the one hand some ideas of “chaos theory” have been presented to the general public, pretending a wide applicability in different areas, but without providing more than a nebulous impression of what “chaos”

really means. On the other hand nonlinear vibrations have found attention in the engineering sciences as can be seen by a growing number of corresponding textbooks [23,24]. Sometimes they are integrated into standard courses on the theory of vibrations (f. i. [25]), but usually they fall victim to a lack of time or, more important, to the student's lack of necessary mathematical prerequisites.

The goal of the short course described here is to close this gap by using “virtual experiments” instead of mathematical derivations to present some important phenomena and a basic physical understanding of nonlinear and chaotic systems. It is part of the multimedia CD-ROM of [11], a version based on the current Java standard can be found at [26].

For a successful comprehension of the course one needs the following prerequisites: In mathematics

- trigonometry,
- basic vector analysis,
- notion of the derivative

which are taught in beginner's mathematics courses, in physics

- decomposition of forces,
- Newton's equation of motion,
- basics of linear oscillations (spring pendulum),

which have been presented in the first physics course and in technical mechanics. The concept of the equation of motion as a differential equation is known from physics, further knowledge about ordinary differential equations, especially methods for finding solutions, are not needed here.

At the end of the course the participants should know the basic types of motion of nonlinear systems:

- harmonic oscillation as an approximation for small amplitudes,
- dependency of the frequency on the amplitude,
- period doubling phenomenon,
- apparently irregular, “chaotic” motion.

They should know the phase space diagram as a tool to investigate nonlinear systems and be able to apply it – together with the Poincare cut – to identify chaotic behavior. Finally they should know how to estimate the dependence of the motion on the initial conditions and its consequences for the prediction of chaotic systems.

The mathematical pendulum with harmonic excitation serves as a fundamental example system. Its equation of motion

$$\ddot{\varphi} + \frac{b}{m} \dot{\varphi} + \frac{g}{l} \sin(\varphi) = \frac{\hat{F}}{ml} \cos(\omega_{ext} t) \quad (1)$$

can be derived explicitly using only the stated mathematical prerequisites. It is examined closely with the given applets, alternating between demonstrations by the lecturer and own experiments done by the students. The applets are used to study the qualitative behavior and for concrete “measurements” leading to quantitative relations, f. i. between the oscillation period and the amplitude for the pendulum without friction.

As a complementing example the Duffing oscillator [27] is presented, given by

its equation of motion

$$\ddot{x} + \lambda \dot{x} - x + \mu x^3 = A \cos(\omega_{ext} t) \quad (2)$$

It approximately describes the motion of an iron pole that is fixed between two attracting magnets. The external excitation is done by horizontally moving the suspension point:

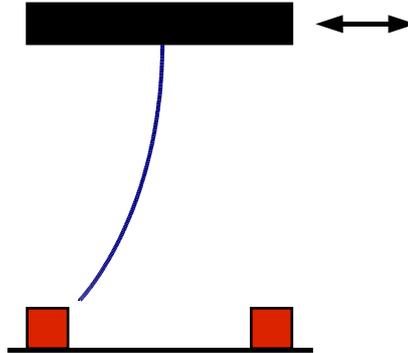


Figure 1 Model of a Duffing oscillator

In contrast to the mathematical pendulum the equation of motion can not be derived without additional information, but its general form can be explained as a slight expansion of Hooke's law. The special advantage of the Duffing oscillator is its very clear series of period doublings on the route to chaos.

To study this system one could of course use applets again, which can in fact be programmed without much effort starting from the already given examples. In this course its behavior is presented in a different way using graphs created with Matlab. This furthermore provides some motivation for a Matlab course which is part of the computer science course of the following semester.

5. Basic Structure of the Applets

All applets that are used in the course have the same basic structure: They consist of an input panel with all input controls, an object panel, which displays the simulated object in an schematic way, and an output panel showing the results of measurements as numbers or graphically. To make this structure immediately evident, each panel type has a characteristic color. Furthermore they are arranged in the same way, if this is possible and reasonable.

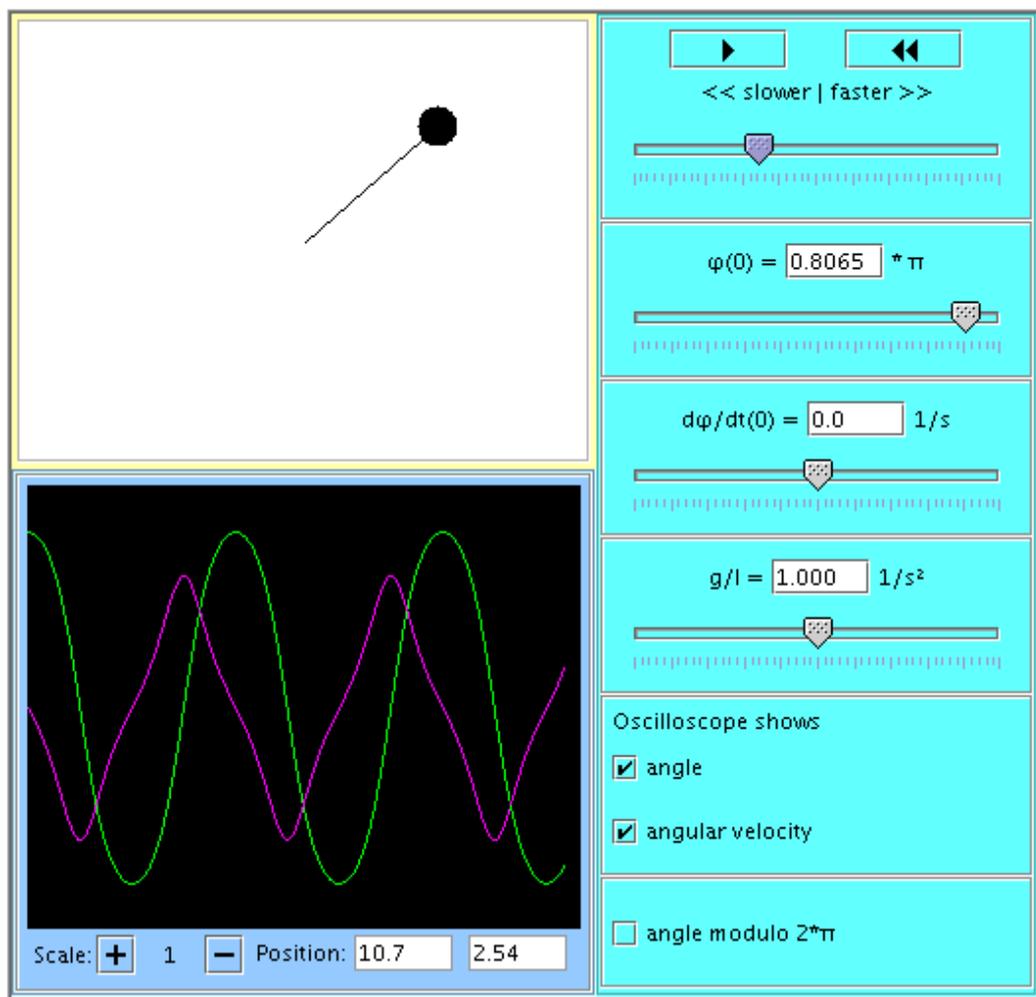


Figure 2 Applet „Mathematical Pendulum“

All manipulations on the simulated system are done via special input elements. The following controls from the PhysBeans library are used for this purpose:

- timer
- numerical input
- selection box

The timer is the basic tool to control the time flow of the simulation. It allows to change the simulation speed, to pause, continue and restart the simulation. By using buttons with icons that are well-known from audio/video recorders, its function is immediately obvious.

All numerical input is done using the same type of control element with a slider for quick qualitative input and a text field for explicit typing-in of the exact number. Of course both fields are coupled and typed numbers are only accepted within the range that is defined by the slider. A short text or symbol explains the physical quantity to change and defines its unit.

Selection boxes are used to choose the curves that are displayed in an oscilloscope. Sometimes they define additional plotting parameters. These boxes replace a legend: Simply switching on and off a box makes evident which color corresponds to which physical quantity.

The output area mainly consists of a simulated oscilloscope that either shows physically interesting quantities as functions of time or one quantity depending

on another one (x-y oscilloscope). Since the magnitudes of the shown parameters can vary largely, the oscilloscope has buttons to increase or decrease the scale in several steps, each time by a factor of two.

Quantitative measurements on the curves can be done easily: After clicking at an arbitrary point in the oscilloscope window, the corresponding coordinates are displayed, using the correct values in the proper SI base units, independently of the chosen display scale. To point out this feature, the cursor changes to a cross-hair symbol when the mouse is over the display area of the oscilloscope. To make it possible that several physical quantities can be displayed within a common numerical scale (assuming base units), the initial values of all input parameters are tuned to guarantee that the corresponding physical quantities have similar magnitudes, when expressed in SI units. Changing system parameters can lead to diverging magnitudes of some curves. In this case a different scale has to be used for each quantity to allow for measurements of comparable accuracy.

In the example applets the object panel shows the motion of the mathematical pendulum in schematic form. A more realistic representation – using three-dimensional or lightning effects - might be desirable, but the necessary programming effort is much larger than the didactic benefit. Moreover the simple animation always reminds of the virtual character of the “experiment”. Some of the applets have no object panel, because it would lead to no new insights and the space is needed for the output area. Sometimes the oscilloscope is assigned to the object panel (by its background color). This means that the displayed curves are – in a more abstract view point - the proper object of measurements.

Due to the consistent use of identical base elements with layouts of similar structure the applets can be used without additional instructions, a more playful approach leads to immediate success. A single exception might be the measurement with the oscilloscope: This basic mechanism probably has to be explained once. The emphasis on immediate intelligibility is the reason that the object panel has no direct manipulation facilities. For instance the initial position of the pendulum can only be changed with an input element, not by direct dragging of the pendulum image with the mouse, a technique that is used in many applets [28]. This limitation corresponds to the situation of many present-day experiments: After an experiment is set up, one usually has no direct contact to the measurement object itself. The above mentioned remote experiments use a similar approach by their very nature.

6. Description and Usage of Selected Applets

In this section we will present six applets that have been used extensively during the course. Besides the description of the applets the focus will be on their applicability and the corresponding teaching goals.

6.1. Mathematical Pendulum

This applet simulates the mathematical pendulum without friction and driving force. Input parameters are the initial conditions for the angle φ and the angular velocity $\dot{\varphi}$ and the only system parameter g/l . For the sake of lucidity changing g/l does not influence the graphical representation of the pendulum. This could

be interpreted as changing g instead of l . The oscilloscope shows the time functions of angle and angular velocity, with the additional possibility to reduce the angle to a basic interval of $-\pi$ to π .

As a first step the applet is used to remind of the basic features of linear oscillators. For this purpose the instructor demonstrates a simple simulation run with a small value of $\varphi(0)$, which the students identify as (approximate) harmonic oscillations. Next the students build small groups to experiment with the applet and to find the functional form of the dependence of the period T on g/l (with all other parameters fixed). The instructor could recapitulate here the relationship between the angular frequency ω and the period T and add the “experimental” hint that a measurement over several periods leads to a better accuracy. Usually at least one group finds the basic relationship

$$T = 2\pi \sqrt{\frac{l}{g}} \quad (3)$$

which can then be derived by linearizing the sine function.

Now the different kinds of movement for larger initial values are studied. During the course they are demonstrated and the students are asked to comment on their observations. The increasing influence of the non-linearity can best be seen by looking at the angular velocity, while the graph of the angle looks rather “harmonic” for a while. Reaching initial angles near 180° one can discuss the question, how realistic the simulation is: A “real” pendulum wouldn’t swing but simply fall down, so the string is more a kind of “massless pole”. The dependency of the period on the amplitude has now become obvious. Adding an initial angular velocity leads to a looping movement, which can be the starting point of a discussion, whether it is sensible to use angles larger than 360° .

Finally the students experiment with the applet to measure the relationship between the period and the amplitude and display their results graphically. If time and the mathematical skills of the students allow, the instructor can derive the approximation formula

$$T(\varphi_0) \approx T_0 \left(1 + \frac{1}{16} \varphi_0^2 \right) \quad (4)$$

using the method of harmonic balance [23], and compare it with the experimental results.

As a result of this lecture the students can name some qualitative differences between the motion of linear and nonlinear oscillators and quantify it using the period-amplitude relationship.

6.2. Representation in Phase Space

In the next step the mathematical pendulum with viscose damping is studied. A new applet (not shown here) is used to demonstrate the effect of the damping and to compare its behavior to the already known linear case.

Now the phase space diagram is introduced as an important tool for the further analysis of oscillatory motion. The students are asked to sketch some phase

curves for the known linear systems and to discuss in small groups how analogous graphs look for the mathematical pendulum. These results are verified or corrected using the next applet:

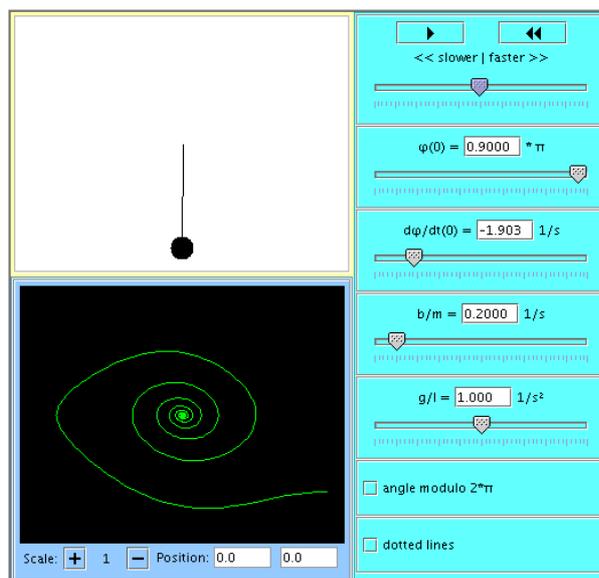


Figure 3 Applet „Phase Space Representation“

It is very similar to the last one, having an additional input element for the damping coefficient b/m . The important difference is the x-y oscilloscope for the display of the phase diagram, i. e. $\dot{\varphi}$ as function of φ . The graphs can optionally be shown in dotted form, which is useful for some of the following investigations.

Using this applet the basic types of graphs for systems with and without damping are demonstrated. To get a better overall picture of the phase space behavior, it is possible to display several curves at once by changing the initial conditions during a simulation run. For this purpose the curves have to be displayed with the dotted option, otherwise one gets distracting lines connecting the end of one curve with the beginning of the next one. The students should realize that curves in phase space don't cross and find an explanation by themselves, using hints from the instructor if necessary.

Analyzing the curves the students should find the idea of an attracting point in phase space, the instructor adding the notion of a repelling fixed point. In the next experiment (as always done in small groups) the students determine a basin of attraction and compare their results with and without reduction of the angle modulo 2π . They now have a basic understanding of all the properties of phase curves that are needed in the following.

6.3. Driven Pendulum

The next applet simulates the mathematical pendulum with harmonic driving force, as given by the following equation of motion (in dimensionally reduced form):

$$\ddot{\varphi} + \lambda \dot{\varphi} + \sin(\varphi) = A \cos(\omega_{ext} t) \quad (5)$$

It contains the usual input controls for the initial conditions and the three parameters λ , A and ω_{ext} . The oscilloscope shows the angle, the angular

velocity and the driving force, where the angle can again be reduced modulo 2π . In the object panel the driving force is represented by an arrow that is attached to the pendulum mass. Length and direction of the arrow show the momentary value of the force.

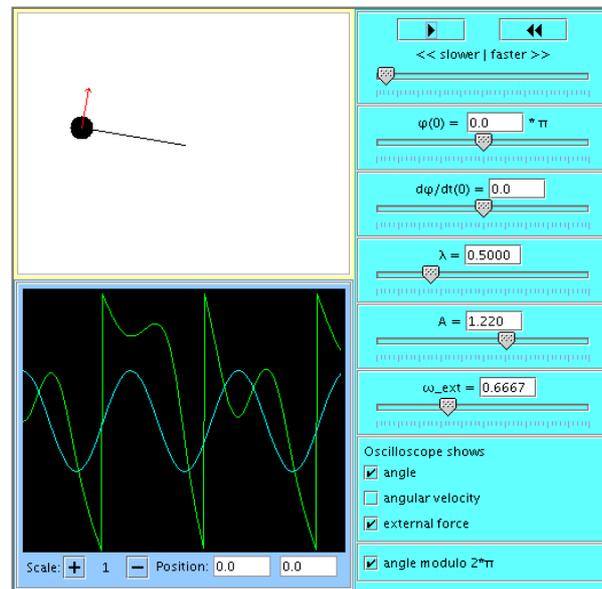


Figure 4 Applet „Driven Pendulum“

The new system is introduced with a discussion how it could be realized in an experiment. Basic problem is the method to transfer a harmonic force to the pendulum, f. i. with a rubber string. Alternatives like a periodic motion of the suspension point are discussed briefly, pointing out possible changes in the equation of motion, without writing down the according formulas explicitly. The equation (5) used here is written down and possibly derived from equation (1).

This is the first example showing chaotic behavior. To understand the complex motion, the two parameters ω_{ext} and λ are fixed in the following (useful values being $\lambda=0.5$, $\omega_{ext}=0.6667$) and the results of changes of the amplitude A are demonstrated. The value $A = 1.0$ basically reproduces the behavior of the linear oscillator, for $A = 1.07$ the period doubling phenomenon appears, and with $A = 1.22$ one has reached the chaotic regime. It is difficult to identify the chaotic oscillations just by looking at the angle modulo 2π . Typically the students try to identify patterns and suspect a larger period. If one focuses on the number of completed turns, the “aimless” and erratic character of the motion gets more apparent.

For the following experiments each group of students gets their own values of λ and ω_{ext} with the task to find the type of motion for varying values of A and to look at the dependence on the initial conditions. At the end all results are compiled in the plenum, where they are critically reviewed. The students now have a basic understanding of the different kinds of motion in chaotic systems. Furthermore they have realized that the space-time diagram is not well suited for such investigations.

6.4. Driven Pendulum in Phase Space

To study the phase space behavior of the driven pendulum an applet is used that is basically identical to the former one, except for the x-y oscilloscope.

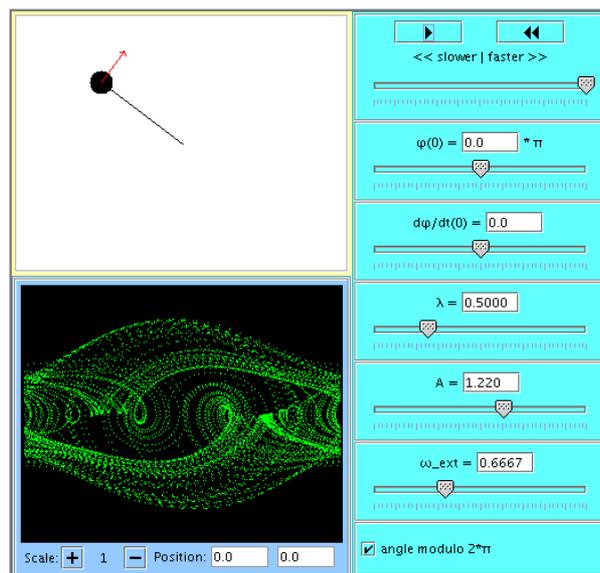


Figure 5 Applet „Driven Pendulum in Phase Space“

At first the standard parameter values are used to demonstrate the behavior of the system in phase space. The phase diagram makes the differences between the different types of motion more apparent: Simple oscillations for small amplitude A lead to an attractor in the form of a “deformed ellipse”, an attractor consisting of two overlapping curves is typical for a system with doubled period. To analyse the chaotic behavior one starts with a small scaling factor of the oscilloscope and without reduction of the angle. This clearly shows the random movement between regions with a different number of completed turns. Repeating the experiment with an angle that is reduced to 2π and a normal scale one gets the well-known “strange attractor”. In a guided discussion the students now try to find the principal difference between this attractor and the non-chaotic phase portraits. The instructor summarizes the discussion and introduces the notion of “non-integral dimension”, without giving a precise definition. This could be done in a later lecture concentrating on fractals.

Finally the students repeat their parameter studies from the last lecture, now using the phase space applet, and correct or extend their former results. This experience immediately shows the importance of the phase diagram in analyzing chaotic systems. In the following compilation of results regions of non-chaotic behavior appear for strong driving forces. This illustrates the complexity of the chaotic system in parameter space as well as in the time domain.

6.5. Poincare Section

If only those points are shown in the phase diagram that correspond to a given fixed phase angle α of the harmonic excitation, a subset of the complete phase portrait is shown that is called a Poincare section. The next applet allows to enter the three system parameters and the phase angle α and displays the resulting phase image in an x-y oscilloscope occupying the object panel. The interpretation of the result is made difficult by the first points which describe the transient oscillations before the attractor is reached (approximately). To get rid of these points the applet allows to enter a delay time during which they are suppressed. For larger delay times this leads to a starting phase during which apparently nothing happens. Therefore the current simulated time is displayed in

the output panel so that the user can easily estimate when the first points will appear.

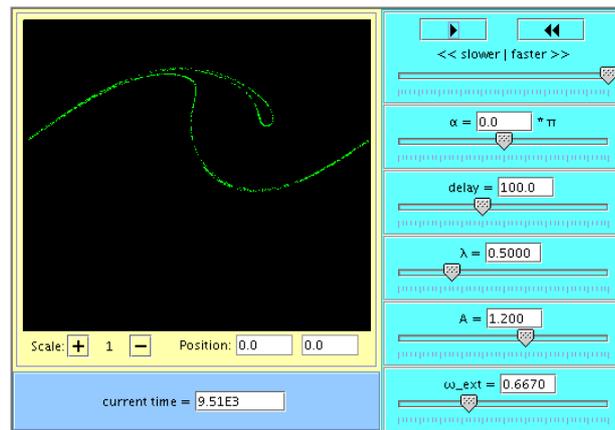


Figure 6 Applet „Poincare Section“

After a short explanation of the function of this applet the students discuss how the basic motion types will appear as Poincare sections. Subsequently the results are demonstrated for the standard values of the parameters: A simple oscillation leads to a single point, period doubling to two points, further doublings to four etc. The strange attractor shows up in the Poincare section as a pattern of many points with interspersed small line parts. Again this brings up the question of the dimension of the curve.

The students now control the results of their previous experiments with this applet. The choice of the delay time needs special care, it has to be large enough to skip the transient phase, especially for small values of the friction coefficient λ . Another point to study is the influence of the phase angle α . After some tests it should be clear that the complete image of the attractor is the combination of all Poincare sections for the different values of α .

With these applets and experiments the students now know the basic types of motion of chaotic systems and some tools to identify them.

6.6. Dependence on the Initial Conditions

The final topic of this course are the consequences of chaotic behavior for the ability to make predictions. The corresponding applet simulates two identical driven pendulums, which start with slightly different initial conditions. Its input values are the three system parameters and the two initial angles, it displays the angles of both pendulums and their difference as functions of time. The oscilloscope occupies the whole object panel, it is rather large to increase the accuracy that is principally limited by the pixel size.

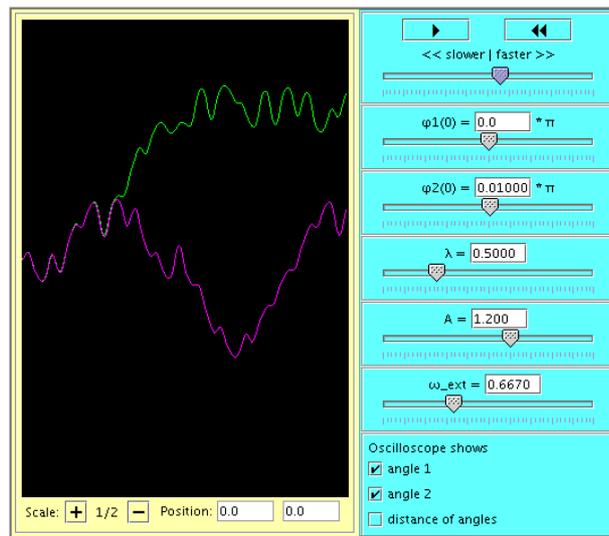


Figure 7 Applet „Initial Conditions“

An alternative applet could show both pendulums and a smaller oscilloscope. Such an applet is in preparation, it makes the actual measurement setup more intuitive. For the student's experiments the more precise version will be used.

To get a qualitative overview of the behavior the students start by studying the divergence of the curves for different types of motion and different initial conditions. They get the following results:

- For a simple periodic oscillation the two curves converge (at least modulo 2π) even for very different initial conditions
- If the oscillations have a doubled period, the pendulums can have different motions, but the difference remains small after a transient period. Making the difference of the initial conditions small enough again results in synchronous motion.
- In the chaotic regime both curves diverge strongly after a certain time. For smaller difference of the initial conditions this time grows, but the divergence phenomenon remains.

The instructor now mathematically describes the observed divergence even for very similar initial conditions by an exponential relation and defines the Lyapunov exponent. The students then use the applet to measure the time until the difference between the two curves reaches a certain fixed value and repeat this experiment for smaller differences in the initial conditions. With these values they can estimate the Lyapunov coefficient of the pendulum.

Finally the consequences of these results for predictions of chaotic systems are discussed leading to the definition of the prediction horizon. Using the results of the previous experiments the students estimate how long one can predict the motion of a driven pendulum taking into account the limited precision of the computer arithmetic. Some results concerning the principal impossibility to make predictions in the large (f. i. number of complete revolutions to the right and left) are explained, as well as new kinds of qualitative predictions (like “system stays on the attractor”).

7. Conclusions

Using applets for the presented course has several advantages:

- Listening to the lecturer is supplemented by own activity.
- Quantitative relations are found using experimental methods instead of formal theoretical reasoning.
- The students play an active part in finding the results, they are working as researchers.
- A current physics topic can be presented, which arises the student's curiosity.

After the experience of the previous physics courses which had been presented in a more conventional lecturing style the students were skeptical about the idea of working actively in small groups. Another problem regularly shows up, when working with computers: A "PC expert" grabs mouse and keyboard and dominates the activities. But after a short time it became clear that general computer experience doesn't help here, and the students began to interact lively, f. i. in discussions how to proceed in a systematic way to make a series of measurements.

In a subsequent course one could examine the causes of chaotic behavior. Important topics are the properties of multi-dimensional functions with several Lyapunov exponents and the definition of fractals. Applets can be used here not only for the simulation of dynamical systems, but to help explain some of the abstract concepts, f. i. to illustrate the iteration of fractal producing maps like the baker map or Koch curve.

Of course applets can be used in many different areas of physics education, as can be seen by the numerous examples found in the internet [9,12,13]. The basic problems are the huge amount of work and the profound Java programming knowledge needed to create such applets. To make the most out of this effort one should use the internet to provide categorized and commented collections of applets that are easy to download and use in physics courses [14]. Another approach is the development of free class libraries for physics applets that can support the programming of new applets substantially.

8. References

1. Gosling, J., Joy, B. and Steele, G.L. *The Java Language Specification*. Boston: Addison Wesley Professional (2000).
2. Chan, P., Lee, R. and Kramer, D., *The Java Class Libraries, Volumes 1, 2 & Supplement*. Boston: Addison Wesley Professional (1998, 1997, 1999).
3. Sun Microsystems, Inc., Java Plug-In Home Page. <http://java.sun.com/products/plugin/>
4. Lea, D., *Concurrent Programming in Java: Design Principles and Pattern*. Boston: Addison Wesley Professional (2000)
5. Walrath, K. and Campione, M., *The JFC Swing Tutorial*. Boston: Addison-Wesley Professional (1999)
6. National Institute of Standards and Technology, *JavaNumerics*. <http://math.nist.gov/javanumerics/>

7. Sun Microsystems, Inc., *NetBeans Home Page*. <http://www.netbeans.org/>
8. Quinn, A., *JavaBeans Tutorial*. <http://java.sun.com/docs/books/tutorial/javabeans/index.html>
9. Junglas, P., *PhysBeans - A JavaBeans Package for Physics Simulation*. In Preparation, cf. <http://www.peter-junglas.de/fh/physbeans/index.html>
10. Bauer, W., Bennenson, W. and Westfall, G., *CliXX Physik (CD-ROM)*. Frankfurt am Main: Harri Deutsch (1999).
11. Stöcker, H., *Taschenbuch der Physik mit CD-ROM*. Frankfurt am Main: Harri Deutsch (2000).
12. PhysicsWeb, *Interaktive Experimente zu Chaos*. http://physicsweb.org/resources/Education/Interactive_experiments/Chaos/
13. Loviscach, J., *Neue Medien in der Lehre, Linksammlung zu Lernsoftware*. http://www.weblearn.hs-bremen.de/home_loviscach/Public
14. Project „physik multimedial“, Link list database. <http://www.physik-multimedial.de/lili/golili/lilierw.php>
15. Elmer, F.-J., *The Pendulum Lab*. <http://monet.physik.unibas.ch/~elmer/pendulum/index.html>
16. Fraser, B., *The Nonlinear Lab*. <http://www.apmaths.uwo.ca/~bfraser/index.html>
17. The MathWorks, Inc, Matlab® <http://www.mathworks.com/>
18. Tiedt, R.-P., *Unterstützung der Mathematikausbildung für Ingenieure durch Nutzung von Matlab*. Global J. of Engng. Educ., 5, 3, 283 - 287 (2001).
19. Waterloo Maple, Inc., Maple®. <http://www.maplesoft.com/>
20. Schramm, T., *Computeralgebrasysteme als Integrationswerkzeuge im Mathematisch-naturwissenschaftlichen Unterricht. Das Mathematische Pendel - Eine Fallstudie*. Global J. of Engng. Educ., 5, 3, 289 - 298 (2001).
21. Georgiev, G.S., Roth, H., Stefanova, S., Georgiev, G.T., Stoyanov, E. and Rösch, O., *How and why to build and use virtual laboratories*. World Trans. Engng. Techn. Educ. 1, 2, 191-196 (2002).
22. Roth, H., Rösch, O., Kuhle, J., Prusak, A., Gonzalez, A.H., Georgiev, G., Lehov, G. and Stefanova, S., *Remote laboratories for experiments in mechatronics*. Proc. 6th Baltic Region Seminar on Engng. Educ., 75-78 (2002).
23. Kapitaniak, T., *Chaos for Engineers. Theory, Applications, and Control*. Berlin: Springer (2000).

24. Baker, G.L., Gollup, J.P., *Chaotic Dynamics*. Cambridge: Cambridge University Press (1996).
25. Magnus, K. and Popp, K., *Schwingungen*. Stuttgart: Teubner (1997).
26. Script „Physik 4“ on the homepage of the author. <http://www.peter-junglas.de/fh/vorlesungen/physik4/html/index.html>.
27. cf. [25], S. 382f
28. z.B. Hwang, F.-K., *NTNU Virtual Physics Laboratory*. <http://www.phy.ntnu.edu.tw/java/Pendulum/Pendulum.html>

9. Biography



Peter Junglas is born in 1959. He studied physics in Hannover and Hamburg specializing in mathematical physics. After the diploma in 1985 he made his Ph. D. with Prof. Buchholz at the Univ. Hamburg in 1987 with a thesis about general quantum field theory. Subsequently he worked two years in this area as assistant at the Univ. Göttingen.

In 1989/90 he was at the MPI for Aeronomy in Katlenburg/Lindau in the group of Horst-Uwe Keller, where he was engaged in image analysis and got a solid training as system administrator.

The next years until 2000 he was working at the computing center of the TU Hamburg-Harburg, at last as vice director. His first tasks included the central server hardware and system software, mainly for the vector and parallel computers. Later he was responsible for the consulting team and the course program. His own interests were scientific computing and parallel programming where he made a point of using standards based techniques like PVM, MPI or OpenMP.

Since 2000 he is professor of physics and computer science at the department of mechanical engineering of the Private University of Applied Sciences Vechta/Diepholz (FHWT). Besides his courses in fundamental science and engineering he plays an active role in establishing a new applied computer science program for engineers. His current main interests lie in the development of multimedia modules for science teaching and in the broad application of simulation techniques. In addition he is engaged in knowledge transfer out of the university, f. i. by giving and organizing public talks on current science topics.